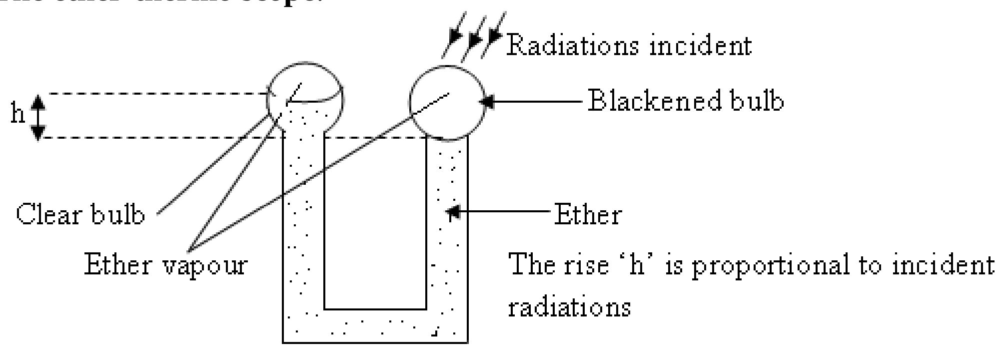
S.6 PHYSICS

P510/1 HEAT (CONTINUED……)

(a) **The ether-thermo scope**.



- A mixture of air and ether vapour is trapped in a tube partly filled with liquid ether.
- When infrared radiations fall on the apparatus, the liquid ether rises into the clear bulb while the level falls in the blackened bulb.
- This is because the radiations that fall on the apparatus is absorbed more by the blackened bulb.
- The rise h in the liquid ether in the clear bulb is proportional to the incident radiation.

BLACK BODY

A black body is body which absorbs all radiations incident on it and reflects or transmits none.

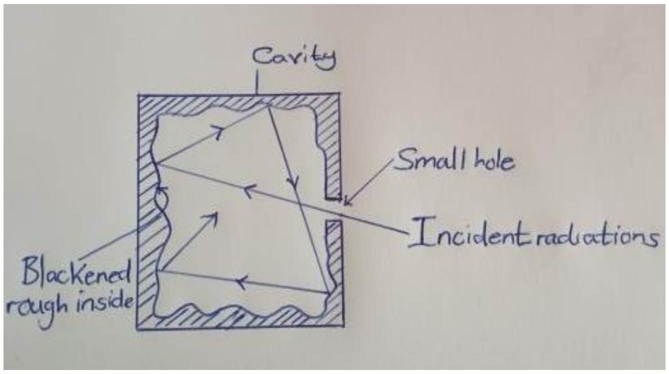
A black body radiation is an electromagnetic radiation emitted by a body solely due its temperature i.e. energy emitted depends on body's temperature.

Black body radiator is a body which emits radiations which are characteristics of its temperature and does not depend on the nature of its surface.

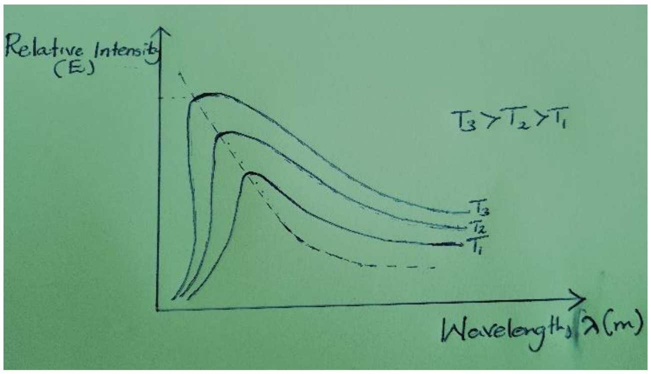
Note: The radiations emitted by a black body are called ***temperature radiations or black body radiations*** because they depend on the temperature of the body

Approximation of black body

- This is done by punching a small hole in an enclosure whose inside walls are rough and painted black.
- At each reflection inside the cavity, a high percentage of the radiations is absorbed and eventually all radiations are absorbed after multiple reflections as shown below.



Distribution of black body radiation



Description of the main features of the curve

- For all wave lengths, an increase in temperature causes an increase in energy of emission.
- The wavelength at which maximum intensity occur shifts to the shorter wavelength as temperature is increased.
- At a given temperature, the energy is not uniformly distributed in the radiation spectrum of a body i.e. energy for different wave lengths is different.
- At a given temperature, the energy of emission (relative intensity) has maximum value for a particular wave length $\lambda_{\max}$.
- The area under each curve represents the total energy emitted for a complete spectrum at a particular temperature.

Change of Colour in relation to black body radiation

- Relative intensity increases with increase in temperature and $\lambda_{\max}$ decreases as temperature is increased.
- The color of the metal being heated depends on the position of $\lambda_{\max}$ in the visible spectrum (**ROYGBIV**) i.e. a body is red if $\lambda_{\max}$ is in the red region of visible spectrum, yellow if $\lambda_{\max}$ is in the yellow region of the visible spectrum or white when $\lambda_{\max}$ is in the middle of the visible spectrum and finally blue when $\lambda_{\max}$ is in the blue spectrum visible region.

NB:

The center of fire scene appears white because;

The scene of fire is hottest at the center. Since intensity of emitting black body is directly proportional to its temperature yet intensity increases rapidly for shorter wave lengths, At the center of fire scene, very many radiations whose wave lengths are in the visible spectrum are emitted. A mixture of these radiations of varying wave lengths constitutes the whiteness of the center of the fire scene.

Question

Explain why welders should put on glasses.

A welder puts on dark glasses which absorb ultraviolet radiations which destroys the retina. This cuts down the intensity of light.

Laws of blackbody radiation

1. **Wein's displacement law** states that as the temperature increases, the maximum intensity of the emission shifts (is displaced) towards the shorter wave length.

Note:

This law can also be stated as the wavelength $\lambda_{\max}$ of radiations emitted by a black body at maximum intensity is such that $\lambda_{max}T = constant$ (the constant 0.002892mK is called Wein's constant and T is the body's absolute temperature.)

2. **Stefan's law (Stefan Boltzmann's law) states** that the total power emitted per unit area of a perfect radiator is proportional to the fourth power of its absolute temperature. i.e.

$$\frac{P}{A} \propto T^4$$
$$P = \sigma A T^4$$

Where σ is Stefan Boltzmann's constant i.e. $\boldsymbol{\sigma = 5.67\text{x}10^{-8}\text{W}m^{-2}K^{-4}}$

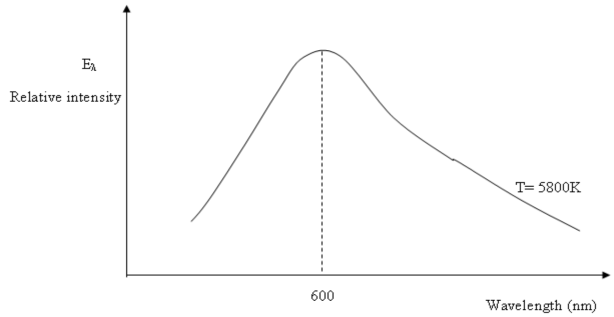
Example

1. Use the figure below to find $\lambda_{\max}$ which corresponds to curves with peaks for;

i) radiation in the sun's core where the temperature is 15×10^{6}K

ii) radiation in an interstellar space whose temperature ≈ 2.7K

(a) Name the radiations emitted in each case.



Solutions:

Using Wien's displacement law $\lambda_{\max}T = \text{Constant}$

(i) At T=15×10^{6}K

$$\lambda_{max}x15x10^6 = 600x5800$$
$$\lambda_{max} = 0.232nm, \quad X - rays$$

(ii) At 2.7K

$$\lambda_{max}x2.7 = 600x5800$$
$$\lambda_{max} = 1.3mm, \quad Infrared\ radiations$$

Emissive power and emissivity

Emissive power is the amount of heat energy emitted per second from unit area of the radiating surface. It is denoted by **e**. it depends only on the nature of the surface and its temperature.

***Emissivity* (ε):** is the ratio of its emissive power to that of a perfect radiator (black body) at the same temperature. i.e.

$$\boldsymbol{\varepsilon} = \frac{\textbf{Total power radiated per square meter of any body}}{\textbf{Total power radiated per squared meter of a black body}}$$

Irradiance is the ratio of total power emitted by a body at equilibrium temperature "T" to its effective surface area of emission. i.e.

Irradiance $= \frac{Total\ power\ radiated\ by\ a\ body\ at\ equilibrium\ temperature\ "T"}{Body's\ effective\ surface\ area\ of\ emission}$ =eσT^4 or σT^4 for perfect black bodies.(since e=1 for a perfect black body)

Example.

1. Calculate the radiant energy per unit area per second of a black body at a temperature of 1200K given that $\sigma = 5.67x10^{-8}Wm^{-2}K^{-4}$. (Ans=1.19x10^5)
2. Calculate the energy radiated per second by a sphere (assumed to be a black body radiator) of diameter 10cm maintained at constant temperature of 727^{o}C. given $\sigma = 5.67x10^{-8}Wm^{-2}K^{-4}$. (1.8x10^{3}W). hint surface area of a sphere is $4\pi r^2$.

3. A cylindrical bulb filament of length 0.5m and radius 1.0×10^{-4}m emits light as black body. 0.4A melts the filament when connected across 240V. Calculate;
   (i) The melting point of the filament
   (ii) The wave length of the radiation emitted at maximum intensity/emission at its melting point.

4. A tungsten filament lamp of 10W lamp at a temperature of 217°C and effective surface area of 0.64cm^2 radiates energy at a rate equivalent to 49% of that radiated by a black body. Calculate Stefan's constant

5. 5.0A flows through a 4.0m long metal filament of an electric lamp of diameter 0.4mm and resistance per centimeter of 22Ωcm^{-1}. Calculate the filament temperature and frequency of the radiation emitted at maximum intensity/emission given that the body radiates as a black body of emissivity 1.0.

6. A closed cube of side 1.0cm has a grey surface which radiates 50% as a black body at 700℃. Calculate;

(i) Power radiated by the cube

Solutions:

Power = $0.5x5.67x10^{-8}x6.0x10^{-4}x\ (973)^4$ = 15.33W

(ii) The radius of a spherical black body which would radiate the same power calculated in 10(i) above when the body it is at 300℃.

Solutions:

15.33 =$5.67x10^{-8}x\ 4\ x\ \pi\ x\ R\ x\ (573)^4$

$$\mathbf{R} = \sqrt{\frac{\mathbf{15.33}}{\mathbf{5.67x10^{-8}x4x\pi x(573)^4}}} = \mathbf{0.014m}$$

7. A cylindrical electric element of length 25cm and diameter 1.5cm is rated 1000W. Determine the filament's equilibrium temperature if it is assumed to behave like a black body.

NB: Leave space after every example for calculation.

Prevost's theory

It States that when the temperature of a body is constant, then the body is losing heat by radiation and gaining by absorption at equal rates.

A body is said to be at equilibrium temperature when its rate of absorption is equal to its rate of emission i.e. the total power leaving the body per square meter must be equal to the total power falling on it.

Radiations inside a constant temperature enclosure

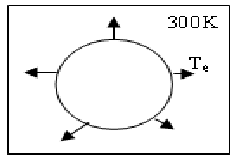
When a perfectly black body of surface area "**A**" at temperature T_1 is enclosed in an enclosure at higher temperature T_2 such that $T_2>T_1$, its net heat gain is given by $P_g = \boldsymbol{\sigma}A\ (T_2^4 - T_1^4)$ and if the body's temperature was higher than that of the enclosure such that $T_1>T_2$ then its net heat loss to the enclosure is given by $P_L = \boldsymbol{\sigma}A\ (T_1^4 - T_2^4)$.

Examples

1. A solid copper sphere of diameter 20mm is cooled to a temperature of 500K and is then placed in an enclosure maintained at 300K. Assuming that all the exchange of heat is caused by radiation, calculate the initial rate of temperature fall if the sphere is assumed to emit like a blackbody.

Solutions:

Assume $\rho_{Cu} = 8930Kgm^{-3}$; enclosure to have the same shape of the sphere, s.h.c of copper is $370JKg^{-1}K^{-1}$.



By principle; the net of heat loss by the body $= P_e - P_b = \delta A\left(T_e^4 - T_b^4\right)$

But by definition this rate of heat loss is given by $= mc\dfrac{\Delta\theta}{\Delta t}$

$$\therefore 5.67*10^{-8}*4\pi*\left(1.0*10^{-2}\right)^2\left(500^4 - 300^4\right) = \frac{4}{3}\pi\left(1.0*10^{-2}\right)^3*8930*370*\Delta\theta$$

$$\Rightarrow \frac{\Delta\theta}{\Delta t} = 0.28Ks^{-1}$$

2. A metal sphere of diameter 1.0×10^{-2}m is cooled to a temperature of 250K and is then placed in an enclosure maintained at 400K. Assumed that all heat exchange is by radiation to calculate the initial rate of rise of temperature of the sphere, assume the blackbody to be similar to the sphere ($\rho_{metal} = 7.20*10^3 kgm^{-3}$, s.h.c of metal is $350Jkg^{-1}K^{-1}$and $\delta = 5.70*10^{-8}Wm^{-2}K^{-4}$)

Solar constant

Solar constant ($S=\sigma T^4$) is the sun's radiant power received on a squared meter of the outside surface of the earth's atmosphere which is perpendicular to the direction of the incident radiations at the sun-earth mean distance. From $P = A\sigma T^4 \Longrightarrow S = \frac{P}{A} = \sigma T^4$. Suppose the power radiated by the sun is $P_s = 4\pi r_s^2\sigma T_s^4$. If the Earth is at a distance R from the Sun, then solar power falls on the total surface area $4\pi R^2$. Solar power falling on a unit surface area is given by $\frac{P_s}{A} = \frac{4\pi r_s^2\sigma T_s^4}{4\pi R^2}$.

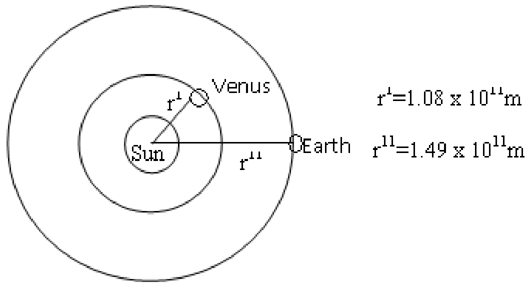
Example

1) The mean distance from the sun to the earth is 1.49×10^{11}m and from the sun to Venus is 1.08×10^{11}m. The absorption rate of the sun and the earth is the same and the earth's solar constant is $S=1.4KWm^{-2}$. Calculate;

i) The solar constant of the Venus.

ii) The equilibrium temperature of Venus assuming that it absorbs and radiates as a black

Solutions:



i) Power absorbed by the earth from the sun is $S * 4\pi r^{11^2}$

Power absorbed by the Venus is $S^1 * 4\pi r^{1^2}$

At equilibrium temperature the rate of absorption by the two planets are equal

$\Rightarrow S * 4\pi r^{11^2} = S^1 * 4\pi r^{1^2}$

$$\therefore S^1 = \frac{1.4*10^3(1.49)^2}{(1.08)^2} = 2.66*10^3 Wm^{-2}$$

ii) By definition, solar constant $S^1 = \delta T_o^4$

$$T_r = \left(\frac{2.66*10^3}{5.67*10^{-8}}\right)^{1/4} = 465K$$

2. The solar constant which is the energy arriving per second at the Earth from the sun is about 1400Wm^{-2}. Estimate the surface temperature of the Sun, given that the Sun's radius is 7x10^{5}km, the distance of the sun from the Earth is 1.5x10^{7}km and staefan's constant is 5.7x10^{-8}Wm^{-2}K^{-4}.

Solution

$$\frac{P_S}{A} = \frac{4\pi r_S^2 \sigma T_S^4}{4\pi R^2} = 1400 \Rightarrow T_S^4 = \frac{1400}{\sigma}\left(\frac{R}{r}\right)^2 \Rightarrow T_S = 5795K$$

Surface temperature of the Earth

This can be estimated on the assumption that the Earth is in radiative equilibrium with the sun. This assumption means that,

Power radiated by the Earth = Power received by the Earth from the Sun

Power radiated by the sun $P_S = 4\pi r_S^2 \sigma T_S^4$

The geometrical projection of the sun on the Earth is a circle whose radius is the radius of the Earth, r_e. Thus, the effective area on which the power falls at right angles is $\boldsymbol{\pi r_e^2}$. But the total area on the solar power falls is $\boldsymbol{4\pi R^2}$, where R is the distance of the Earth from the Sun. So, the fraction of solar radiation incident on the Earth is given by $\frac{\pi r_e^2}{4\pi R^2}(4\pi r_s^2 \sigma T_s^4)$

Power radiated by the Earth = $4\pi r_e^2 \sigma T_e^4$

For radiative equilibrium, power radiated by the Earth = power received by the Earth,

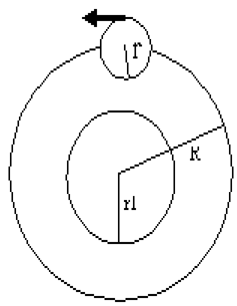
$$4\pi r_e^2 \sigma T_e^4 = \frac{\pi r_e^2}{4\pi R^2}(4\pi r_s^2 \sigma T_s^4) \Longrightarrow T_e = T_s\sqrt{\frac{r_s}{2R}}$$

Also; solar energy = $\frac{\pi r_e^2}{4\pi R^2}$ **x energy radiated by the Sun**

Example

1. Find the equilibrium temperature T_e of the planet earth which revolves around the sun in an elliptical orbit of radius 1.5×10^{11}m (Temperature of the sun temperature is T_s=5800K, Stefan's constant=5.67×10^{-8}Wm^{-2}K^{-4}, sun's radius is 7.0×10^{8}m, emissivity of the sun and earth is 1.0 and earth's radius is $6.4\text{x}10^{6}$m)

Solution



By Stefan's law, the power radiated by the sun $= \delta A T^4$

$$P_{rad} = 5.67*10^{-8}*4\pi(7.0*10^8)^2*(5800)^4 = 4.0*10^{26}W$$

Assuming the area enclosed by the two planets to be spherical

Power transmitted by 1m^2 of this area $= \frac{P_{rad}}{4\pi R^2}$

Power absorbed by the earth from the sun $P_a = \frac{P_{rad}}{4\pi R^2} * A_{earth}$

Where A_{earth} is the effective area of the earth on which radiations from the sun are incident normally. Experiments have proved it that this area is equivalent to area of a circular earth i.e.**A_{earth}= πr^2** "r" is radius of the earth.

$$P_a = \frac{4.0*10^{26}*(6.4*10^8)^2}{4*(1.5*10^{11})^2} = 1.82*10^{17}W$$

Power emitted by the sun $= \delta A T_e^4$

$= 5.67*10^{-8} * 4\pi(6.4*10^6)^2 T_e^4 = 2.92*10^7 T_e^4$

At equilibrium, the power absorbed by the earth from the sun = power emitted by the earth when it is at T_e.

$\therefore 1.82*10^{17} = 2.92*10^7 T_e^4 \Rightarrow T_e = 281.0K$

2. The total power output of the sun is $4x10^{26}$W. Given that the mass of the sun is $1.97x10^{30}$kg and its density is $1.4x10^3$kgm^{-3}. Estimate the temperature of the sun.

Solution

Volume of the Sun $= \frac{mass}{density} = \frac{1.97\times10^{30}}{1.4\times10^3} = 1.407\times10^{27}m^3$. Assuming the sun is spherical, then $1.407\times10^{27} = \frac{4}{3}\pi R_S^3 \Rightarrow R_S = 6.95\times10^8 m$.

Surface area of the sun = $4\pi R_S^2 = 4\times3.14\times(6.95\times10^8)^2 = 6.07\times10^{18}m^2$

Intensity of radiation or solar constant= $\frac{P_S}{A} = \frac{4x10^{26}}{6.07\times10^{18}} = 6.59\times10^7$. Therefore, from

$\frac{P_S}{A} = \sigma T_S^4 \Rightarrow T_S = \left(\frac{6.59\times10^7}{5.67\times10^{-8}}\right)^{\frac{1}{4}} = 5839K$

3. The surface temperature of the sun is 6000K and the Earth orbital radius about the sun is $1.5x10^{11}$m. Calculate the amount of solar energy approaching the Earth from the sun. State any assumption made.
4. Estimate the temperature of the Earth assuming that it is in radiative equilibrium with the sun. (ans:290K)

NB: Leave space after every example for calculation.

Revision Questions.

1. The tungsten filament of an electric lamp has a length of 0.5m and a diameter of 6×10^{-5}m. The power rating of the lamp is 60W. Assuming the radiation from the lamp is equivalent is 80% that of a perfect blackbody radiator at the same temperature, estimate the steady temperature of the filament.

2. The intensity of radiant energy from a blackbody is maximum at a wavelength of 1.5×10^{-6}m, find the corresponding temperature of the blackbody.

3. The resistance of a tungsten wire of an electric lamp at 20°C is 50Ω. At an operating voltage of 240V, the current through the filament is 0.5A. Given that the temperature coefficient of resistance of tungsten is constant and is equal to 5.0×10^{-3}K. Find the;
   i) temperature of the filament at the above operating voltage
   ii) Stefan's constant taking the effective area of the filament to be 0.93cm^2 and assumed a blackbody.

4. The filament of an electric bulb attains a temperature of 1600K when the power supplied to it is 25W.

i) Find the temperature of the filament if the power supply is increased to 60W.
ii) Find the length of the filament at 1600K if the diameter is 5.0×10^{-5}m.
iii) Calculate the difference in the wavelength of the radiation emitted with maximum intensity at the two temperatures (Assume the filament to radiate like a blackbody).

5. A solid copper sphere of diameter 10mm and temperature 150K is placed in an enclosure maintained at temperature of 290K. Calculate, stating the assumptions made, the initial rate of temperature rise of the sphere. ($\rho_{Cu} = 893kg/m^3$, s.h.c of Copper=310$Jkg^{-1}K^{-1}$)

6. (i) Assuming the sun is a sphere of radius 7.0×10^8m at temperature of 6000K; estimate the temperature of the surface of mars if its distance from the sun is 2.28×10^{11}m.
(ii)The energy intensity received by the earth from the sun is 2.4×10^3 Wm^{-2}, calculate the surface temperature of the sun and state any assumptions made

7. (a) (i) Define black body and black body radiation.
(ii)Explain the mechanism of heat transfer through good conductor metals and gases.
(b) Describe the experiment for determining thermal conductivity of a good conductor.
(c) (i) Explain why when determining thermal conductivity of cork, it is thin.
(ii) Assuming the sun to be a sphere of radius 7.0×10^8m at 6000K. Estimate the temperature of planet mars which is at a distance of 2.28×10^{11}m from the center of the sun.

9. (a) (i) Explain why when charcoal is steadily heated, it appears reddish before turning white.
(b) (i) Explain what is meant by a black body absorber.
(ii) Describe how the above body can be realized in practice.
(iii) Account for the fact that metals are good conductors of heat.
(c) (i) State the factors which affect the rate of heat flow through a material.

10. (a) (i) State Wien's displacement law and Stefan's law of black body radiation.
(ii) Show how relative intensity of a black body varies with wave length at different temperatures.
(iii) Explain how the curves in a (ii) above account for the change of color by a metal being heated.
b) (i) Define solar constant and Emissivity
(ii) State Prevost's theory.
(c) Use the figure below to find λ_{max} which corresponds to curves with peaks for;
(i) Radiation in the sun's core where temperature is approximated to 15×10^6K
(ii) Radiation in an interstellar space whose temperature is 2.7k.